Keywords: PJM and SPP energy markets, real-time markets, day-ahead markets, virtual transactions.

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REAL-TIME AND DAY-AHEAD ENERGY MARKETS

Thesis theme:
Էլեկտրաէներգիայի շուկաներում գործարքների ամբողջական պահանջների մոդելավորում

Scientific supervisor:
Վիկտոր ՕՀԱՆՅԱՆ
Ֆիզիկամաթեմատիկական գիտությունների դոկտոր, պրոֆեսոր

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Introduction. Pennsylvania-New Jersey-Maryland Interconnection LLC (PJM) is a regional transmission organization (RTO) in the United States. It is part of the Eastern Interconnection grid operating an electric transmission system serving all or parts of Delaware, Illinois, Indiana, Kentucky, Maryland, Michigan, New Jersey, North Carolina, Ohio, Pennsylvania, Tennessee, Virginia, West Virginia, and the District of Columbia.

PJM, headquartered in Valley Forge, Pennsylvania, was the world's largest competitive wholesale electricity market until the development of the European Integrated Energy Market in the 2000s (see[1]).

Founded as an 11-member tight power pool in 1941, Southwest Power Pool (SPP) achieved RTO status in 2004, ensuring reliable power supplies, adequate transmission infrastructure, and competitive wholesale electricity prices for its members. Based in Little Rock, Ark., SPP manages transmission in fourteen states: Arkansas, Iowa, Kansas, Louisiana, Minnesota, Missouri, Montana, Nebraska, New Mexico, North Dakota, Oklahoma, South Dakota, Texas and Wyoming. Its membership is comprised of investor-owned utilities, municipal systems, generation and transmission cooperatives, state authorities, independent power producers, power marketers and independent transmission companies.

In 2007, SPP began operating its real-time Energy Imbalance Service (EIS) market. In the same year, SPP became a FERC-approved Regional Entity. The SPP Regional Entity serves as the reliability coordinator for the NERC region, overseeing compliance with reliability standards (see[2]).

Virtual transactions in PJM and SPP are bids and offers submitted to take financial positions in the Day-Ahead Market without the intent of delivering or consuming physical power in the Real-Time Market.

Virtual transactions are a valuable component of a two-settlement market such as the PJM and SPP market. They have the ability to mitigate both supply-side and demand-side market power by allowing market participants without physical assets to compete with asset owners and load-serving entities in the market.

Because virtual transactions compete with physical resources in the Day-Ahead Market, they can either displace, or cause additional scheduling of, physical resources and load, including price-sensitive demand bids. These changes in the Day-Ahead Market outcome due to virtual transactions may or may not match what is needed in the Real-Time Market. Regardless, the Day-Ahead Market results, including resource commitments, dispatch and pricing, all are impacted by virtual transactions every day.

A market participant submitting a virtual transaction that clears takes a financial position in the Day-Ahead Market by agreeing to buy or sell energy at a specific location or locations that it then liquidates in the Real-Time Market. This occurs because the energy that is bought or sold in the Day-Ahead Market is not provided or consumed in real-time and creates an imbalance between the markets.

Virtual transactions can benefit the market in several different ways. However, by competing with physical resources in the Day-Ahead Market, virtual transactions can affect how physical resources are scheduled and dispatched, impacting Locational Marginal Price (LMP) and uplift costs.

INCs are offers submitted in the Day-Ahead Market to sell a stated amount of energy at a specified location. From a Day-Ahead Market clearing perspective, these offers can be thought of as equivalent to a generation offer without temporal restrictions (such as startup times and minimum run times). An INC will clear if the day-ahead clearing price for that node exceeds the offer price (see[3]).

Real-Time Markets. In order to explore the Real-Time markets, we have taken RT prices of several large points in SPP markets, that sell energy, at an hourly basis, and we have made an index from their average, that shows the behavior of the RT prices in that

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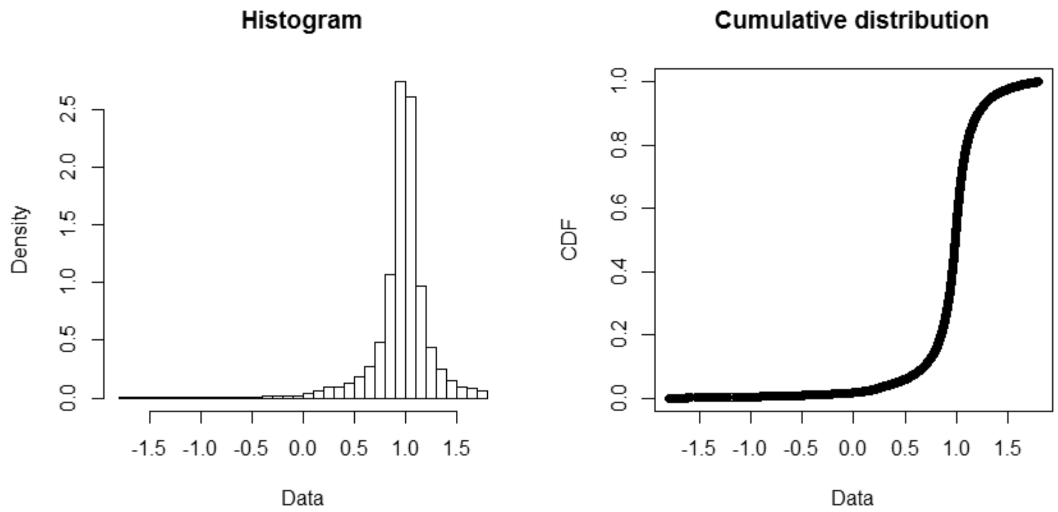
[1] PJM 2016 Annual report, Revolutionary thinking, 2016, pp.11-13
[2] https://www.ferc.gov/market-oversight/mkt-electric/spp.asp
[3] Virtual Transactions in the PJM Energy Markets, PJM Interconnection, 2015, pp.1-5

area. In total, these data are 34983, and the observed points belongs to: "COFFEYVILLE", "BUFFALO DUNES WIND", "CANADIAN HILLS", "CANEY RIVER WINDPOWER", "BLUE CANYON" (see Appendix 1).

Now let us take the prices of our index and remove from them extreme values, that are about 1000. Because of we have 4 years of data, there can be extreme values that have been dependent on something at a given moment and do not have a repetitive character, but can spoil the distribution function of the prices.

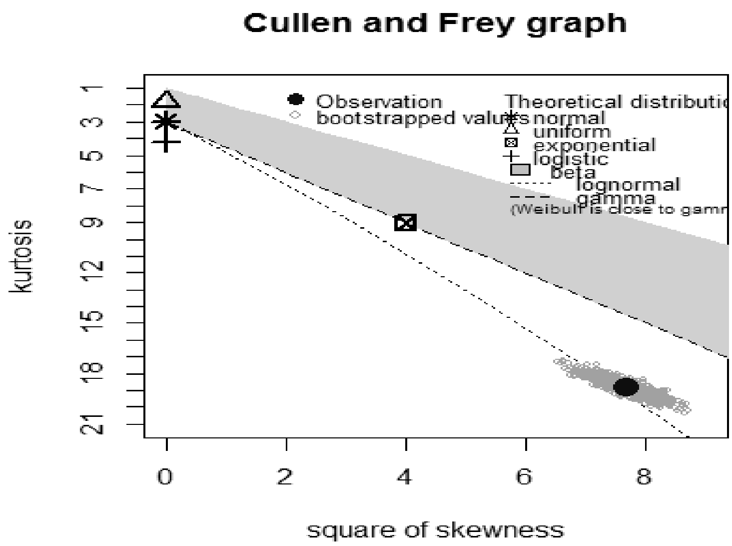
After all let us take the returns of the prices of index, and by the R programming language, see what distribution function will fit them (see Appendix 2).

Figure1.



In Figure 1, it is shown density and distribution function of our data. In Figure 2 Cullen and Frey graph shows us that our observation falls very close to the lognormal distribution. For more practical result, we do the same with bootstrapped values, that values are populated densely, it means that our result is good. In our data we have negative values, but as their weight is very small in the data, the distribution of returns is very close to the lognormal distribution.

Figure 2.



Now let us fit Pearson distribution on our data of returns, and see what will show us R programming language (see[4]).

From Table 1 follows, that our series of returns has Pearson type VI distribution, with $\alpha_1 = 1.43, \alpha_2 = 8.05,\ \gamma = 1.28, \beta = -1.60$ parameters.

Table 1.

[4] http://www.mathwave.com/help/easyfit/html/analyses/distributions/pearson6.html

```
$type
[1] 6

$a
[1] 1.427691

$b
[1] 8.054372

$location
[1] 1.276112

$scale
[1] -1.599713
```

The Distribution Function of Pearson Type VI is:

$$F(x) = I_{(x-\gamma)/(x-\gamma+\beta)}(\alpha_1, \alpha_2), \tag{1}$$

The Probability Density Function is:

$$f(x) = \frac{((x-\gamma)/\beta)^{\alpha_1 - 1}}{\beta\, B(\alpha_1, \alpha_2)(1 + (x-\gamma)/\beta)^{\alpha_1+\alpha_2}}, \tag{2}$$

where B is the Beta Function, and I_z is the Regularized Incomplete Beta Function. And parameters are: $\alpha_1 > 0$, $\alpha_2 > 0$ continuous shape parameters, β continuous scale parameter, γ continuous location parameter.

Using parameters in equation (2), we will have such form of density function for the returns of energy market RT prices:

$$f(x) = \frac{((x-1.28)/(-1.60))^{1.43-1}}{(-1.60)\, B(1.43{,}8.05)(1 + (x-1.28)/(-1.60))^{1.43+8.05}} = \frac{62.5(x-1.28)^{0.43}}{B(1.43{,}8.05)(x-2.88)^{9.48}}. \tag{3}$$

And using them in equation (1), we will have such form of distribution function for the returns of energy market RT prices:

$$F(x) = I_{(x-1.28)/(x-1.28-1.60)}(1.43{,}8.05) = \frac{B_{(x-1.28)/(x-2.88)}(1.43{,}8.05)}{B(1.43{,}8.05)}. \tag{4}$$

So we can formulate a theorem about RT prices of SPP energy markets.

Theorem 1. The returns of Real-Time prices of SPP energy markets have Pearson Type VI distribution with $\alpha_1 = 1.43, \alpha_2 = 8.05,\ \gamma = 1.28, \beta = -1.60$ parameters. And it's distribution function and density function have the forms (3) and (4).

Now let us take RT prices from PJM market large points and fit Pearson distribution on their returns. We take 45000 data, and make an index from that RT prices. In this case, R programming language shows us that our returns of prices have Pearson Type IV distribution.

Table 2.

```
$type
[1] 4

$m
[1] 2.808495

$nu
[1] 0.6873922

$location
[1] 1.05739

$scale
[1] 0.3300014
```

Here we have $m = 4, \nu = 2.81, a = 1.06, \lambda = 0.33$ parameters.

The Pearson Type IV probability density function is given by

$$f(x)dx = k\left[1 + \left(\frac{x-\lambda}{a}\right)^2\right]^{-m} \exp\left[-\nu \tan^{-1}\left(\frac{x-\lambda}{a}\right)\right] dx,$$

where m, ν, a and λ are real-valued parameters, and $-\infty < x < +\infty$ (k is a normalization constant that depends on m, ν and a) (see [5]).

Let us formulate a theorem about RT prices of PJM energy markets.

Theorem 2. The returns of Real-Time prices of PJM energy markets have Pearson Type IV distribution with $m = 4, \nu = 2.81, a = 1.06, \lambda = 0.33$ parameters.

Day-Ahead Markets. In this section, we want to check the dependency of DA and RT prices in SPP and PJM markets, and see whether we can make our predictions based on DA prices or not. To check this, we take a part of the RT prices, and do the regression DA and RT prices. Then we get the other part of the RT prices from regression results. In the end, we will compare the received prices with real prices to check whether our method has produced a good result or not (see Appendix 3).

Firstly, let us define:

a - 2/3 part of given DA prices,

b – 2/3 part of RT prices,

a_0 – other 1/3 part of given DA prices,

b_0 – other 1/3 part of RT prices.

Now let us do the regression between b and a ($b = \alpha + \beta a + \varepsilon$).

Table 3.

```
Call:
lm(formula = b ~ a)

Residuals:
    Min      1Q  Median      3Q     Max 
-371.80   -4.79   -1.27    1.02 1231.15 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.35670    0.40560    5.81 6.33e-09 ***
a            0.86172    0.01346   64.04  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 24.51 on 19998 degrees of freedom
Multiple R-squared:  0.1702,	Adjusted R-squared:  0.1701 
F-statistic:  4101 on 1 and 19998 DF,  p-value: < 2.2e-16
```

Table 4.

```
Call:
lm(formula = b ~ a)

Residuals:
    Min      1Q  Median      3Q     Max 
-623.68   -5.34   -2.97   -0.27 1561.71 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 7.504663   0.282956   26.52   <2e-16 ***
a           0.786027   0.005795  135.63   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 30.82 on 29998 degrees of freedom
Multiple R-squared:  0.3801,	Adjusted R-squared:  0.3801 
F-statistic: 1.839e+04 on 1 and 29998 DF,  p-value: < 2.2e-16
```

Table 3 shows the summary of the regression in SPP markets, and Table 4 shows the summary of the regression in PJM markets, and here we can see the coefficients, and as the p-value is much less than 0.05, we reject the null hypothesis that $\beta = 0$. Hence there is a significant relationship between the variables in the linear regression model of the data set faithful.

So we can now use our significant coefficients to forecast other 1/3 part of the RT prices. Let us define p – our predictions of 1/3 part of the RT prices, and d – difference of real prices and our predictions.

[5] J. Heinrich, “A Guide to the Pearson Type IV Distribution”, University of Pennsylvania, December 21, 2004, pp. 1-2

$$p = \hat{\alpha} + \hat{\beta} \cdot a_0,$$
$$d = b_0 - p\ .$$

Figure 3.

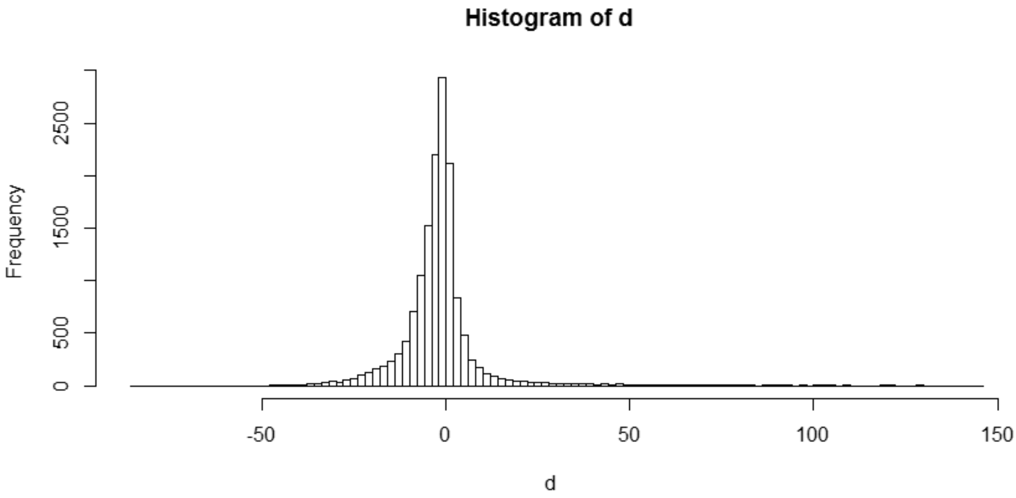
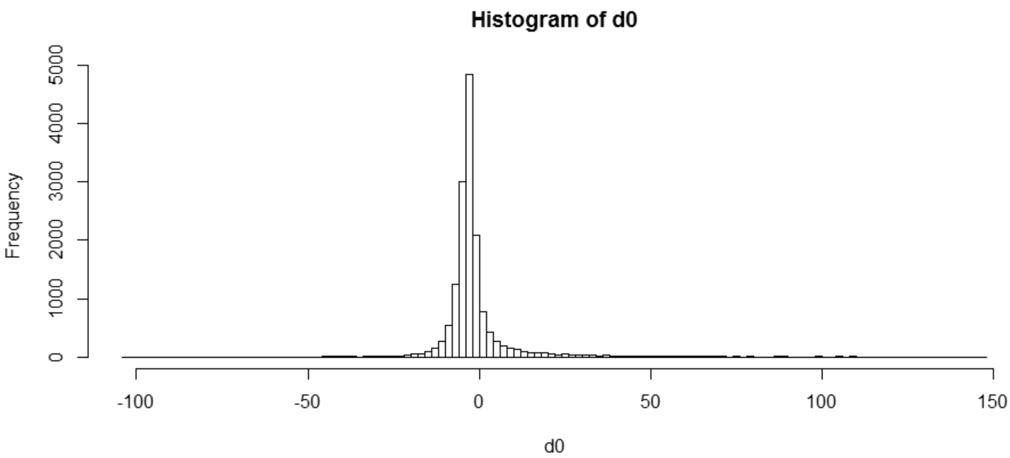


Figure 4.



Histograms in Figure 3 and 4 show that our data of difference has normal distribution in SPP and PJM markets, respectively with parameters that are shown in Table 5 and 6.

Table 5.

```
Fitting of the distribution ' norm ' by maximum likelihood
Parameters:
      estimate Std. Error
mean -1.098458  0.1483830
sd   18.162835  0.1049227
```

Table 6.

```
Fitting of the distribution ' norm ' by maximum likelihood
Parameters:
      estimate Std. Error
mean -1.283515 0.09668075
sd   11.978273 0.06836361
```

The results are good, the means of differences are approximately -1, so the series we get is very close to the real RT prices, so we can say that having DA prices we can add the coefficients from our regression and make a better forecast of RT prices.

References:

1. PJM 2016 Annual report, Revolutionary thinking, 2016, pp.11-13
2. https://www.ferc.gov/market-oversight/mkt-electric/spp.asp
3. Virtual Transactions in the PJM Energy Markets, PJM Interconnection, 2015, pp.1-5
4. http://www.mathwave.com/help/easyfit/html/analyses/distributions/pearson6.html

5. J. Heinrich, “A Guide to the Pearson Type IV Distribution”, University of Pennsylvania, December 21, 2004, pp. 1-2

Appendix 1.

DATE	DALMP	RTLMP
12/1/2013 1:00	28.248	-18.8895
12/1/2013 2:00	26.056	-27.636
12/1/2013 3:00	25.356	-67.9513
12/1/2013 4:00	26.042	16.09486
12/1/2013 5:00	26.426	17.2365
12/1/2013 6:00	26.538	16.9873
12/1/2013 7:00	26.162	11.48516
12/1/2013 8:00	29.866	18.62148
12/1/2013 9:00	36.016	-32.4172
12/1/2013 10:00	35.2	13.55402
12/1/2013 11:00	30.22	16.05234
12/1/2013 12:00	28.312	-102.993
12/1/2013 13:00	25.808	15.22316
12/1/2013 14:00	25.082	-163.917
12/1/2013 15:00	24.214	-348.576
12/1/2013 16:00	24.214	16.0535
12/1/2013 17:00	28.57	20.81952
12/1/2013 18:00	38.14	102.2505
12/1/2013 19:00	57.048	24.22464
12/1/2013 20:00	41.288	26.48666
12/1/2013 21:00	37.564	22.51534
12/1/2013 22:00	32.2	17.26068
12/1/2013 23:00	35.532	-39.4567
12/2/2013 0:00	26.27	6.37716
12/2/2013 1:00	23.704	7.81382
12/2/2013 2:00	21.694	13.95952
12/2/2013 3:00	21.372	14.76116
12/2/2013 4:00	22.23	16.51936
12/2/2013 5:00	23.402	17.29866
12/2/2013 6:00	27.26	19.85416
12/2/2013 7:00	32.5	54.09266
12/2/2013 8:00	35.514	25.38682
12/2/2013 9:00	35.342	21.396

Etc…

Appendix 2.

```
library("readxl")
library("fitdistrplus")
library("actuar")
  library("PearsonDS")
  aaa<- read_excel("C:/Users/Tigran.Piliposyan/Desktop/2.xlsx")
  fff<-c()
  ggg<-c()
  for( i in 1:34983){
   fff[i]<-aaa[[i,2]]
   ggg[i]<-aaa[[i,3]]
  }
```

```
ggg<-ggg[-which(ggg==0)]
bbb<-1:34981
for(i in 1:34981){
  bbb[i]<-ggg[i+1]/ggg[i]
 }
ccc<-bbb[-which(bbb>mean(bbb)+3*sd(bbb))]
ddd<-ccc[-which(ccc< mean(bbb)-3*sd(bbb))]
ccc1<-bbb[-which(bbb>1.8)]
ddd1<-ccc1[-which(ccc1< -1.8)]
hist<-plotdist(ddd1, histo = TRUE,breaks=50)
CulFrey1<-descdist(ddd1,boot=1000)
hist<-plotdist(fff, histo = TRUE,breaks=50)
CulFrey2<-descdist(fff,boot=1000)
pearsonDiagram(max.skewness = 26, max.kurtosis = 24,
        squared.skewness = TRUE, lwd = 2, legend = TRUE,
        n = 301)
x<-empMoments(ddd1)
xx<-pearsonFitM(x[[1]],x[[2]],x[[3]],x[[4]])
```

Appendix 3.

```
library(readxl)
X2 <- read_excel("C:/Users/Tigran.Piliposyan/Desktop/works/Tigran/2.xlsx")
a<-c()
a0<-c()
b<-c()
b0<-c()
for(j in 1:20000){
  a[j]=X2[[j,2]]
  b[j]=X2[[j,3]]
}
for(j in 20001:34983){
  a0[j-20000]=X2[[j,2]]
  b0[j-20000]=X2[[j,3]]
}
x<-lm(b~a)
y<-x$coefficients
p<-c()
for( j in 1:14983){
  p[j]<-y[[1]]+y[[2]]*a0[j]
}
d<-b0-p
hist(d,breaks=100)
```

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